CONFIRMATOIN OF THE σ-MESON BELOW 1 GEV AND INDICATION FOR THE $f_0(1500)$ GLUEBALL

Yu.S. Surovtsev

Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna 141 980, Moscow Region, Russia

D. Krupa and M. Nagy

Institute of Physics, Slov.Acad.Sci., Dúbravská cesta 9, 842 28 Bratislava, Slovakia

On the basis of a simultaneous description of the isoscalar s-wave channel of the $\pi\pi$ scattering (from the threshold up to 1.9 GeV) and of the $\pi\pi \to K\overline{K}$ process (from the threshold to ~ 1.4 GeV) in the model-independent approach, a confirmation of the σ-meson at ~ 660 MeV and an indication for the glueball nature of the $f_0(1500)$ state are obtained.

A problem of scalar mesons is most troublesome and long-lived in the light meson spectroscopy. Among difficulties in understanding the scalar-isoscalar sector there is the one related to a strong model-dependence of information on multichannel states obtained in analyses based on the specific dynamic models or using an insufficiently-flexible representation of states (*e.g.*, the standard Breit – Wigner form). Earlier, we have shown [1] that an inadequate description of multichannel states gives not only their distorted parameters when analyzing data but also can cause the fictitious states when one neglects important (even energetic-closed) channels. In this paper we are going, conversely, to demostrate that the large background (*e.g.*, that happens in analyzing $\pi\pi$ scattering), can hide low-lying states, even such important for theory as a σ-meson [2]. The latter is required by most of the models (like the linear σ-models or the Nambu – Jona-Lasinio models [3]) for spontaneous breaking of chiral symmetry. Since earlier all the analyses of the s-wave $\pi\pi$ scattering gave a large $\pi\pi$-background, it was said that this state (if exists) is "unobservably"-wide. Recently, new analyses of the old and new experimental data have been performed which give a very wide scalar-isoscalar state in the energy region 500-850 MeV [4]. However, these analyses use either the Breit – Wigner form (even if modified) or specific forms of interactions in a quark model, or in a multichannel approach to the considered processes; therefore, there one cannot talk about a model independence of results. Besides, in these analyses, a large $\pi\pi$-background is obtained. We are going to show

that a proper detailing of the background (as allowance for the left-hand branch-point) permits us to extract from the latter a very wide (but observable) state below 1 GeV.

An adequate consideration of multichannel states and a model-independent information on them can be obtained on the basis of the first principles (analyticity, unitarity and Lorentz invariance) immediately applied to analyzing experimental data. The way of realization is a consistent allowance for the nearest singularities on all sheets of the Riemann surface of the S-matrix. The Riemann-surface structure is taken into account by a proper choice of the uniformizing variable. Earlier, we have proposed this method for 2- and 3-channel resonances and developed the concept of standard clusters (poles on the Riemann surface) as a qualitative characteristic of a state and a sufficient condition of its existence as well as a criterion of a quantitative description of the coupled-process amplitudes when all the complifications of the analytic structure due to a finite width of resonances and crossing channels and high-energy "tails" are accumulated in quite a smooth background [1]. Let us stress that for a wide state, the pole position (the pole cluster one for multichannel states) is a more stable characteristic than the mass and width which are strongly dependent on a model. The cluster kind is determined from the analysis of experimental data and is related to the state nature. At all events, we can, in a model-independent manner, discriminate between bound states of particles and the ones of quarks and gluons, qualitatively predetermine the relative strength of coupling of a state with the considered channels, and obtain an indication on its gluonium nature.

In this work, we restrict ourselves to a two-channel approach when considering simultaneously the coupled processes $\pi\pi \to \pi\pi, K\overline{K}$. Therefore, we have the two-channel S-matrix determined on the 4-sheeted Riemann surface. The S-matrix elements $S_{\alpha\beta}$, where $\alpha,\beta = 1(\pi\pi), 2(K\overline{K})$, have the right-hand (unitary) cuts along the real axis of the s-variable complex plane, starting at the points $4m_\pi^2$ and $4m_K^2$ and extending to ∞, and the left-hand cuts, which are related to the crossing-channel contributions and extend along the real axis towards $-\infty$ and begin at $s=0$ for S_{11} and at $4(m_K^2 - m_\pi^2)$ for S_{22} and S_{12}. We number the Riemann-surface sheets according to the signs of analytic continuations of the channel momenta $k_1 = (s/4 - m_\pi^2)^{1/2},\ k_2 = (s/4 - m_K^2)^{1/2}$ as follows: signs $(\mathrm{Im}k_1, \mathrm{Im}k_2) = ++, -+, --, +-$ correspond to the sheets I,II,III,IV.

To elucidate the resonance representation on the Riemann surface, we express analytic continuations of the matrix elements to the unphysical sheets $S_{\alpha\beta}^L$ ($L = II, III, IV$) in terms of them on the physical sheet $S_{\alpha\beta}^I$. Those expressions are convenient for our purpose because, on sheet I (the physical sheet), the matrix elements

$S^I_{\alpha\beta}$ can have only zeros beyond the real axis. Using the reality property of the analytic functions and the 2-channel unitarity, one can obtain

$$\begin{array}{lll} S_{11}^{II}=\dfrac{1}{S_{11}^{I}}, & S_{11}^{III}=\dfrac{S_{22}^{I}}{\det S^{I}}, & S_{11}^{IV}=\dfrac{\det S^{I}}{S_{22}^{I}}, \\ S_{22}^{II}=\dfrac{\det S^{I}}{S_{11}^{I}}, & S_{22}^{III}=\dfrac{S_{11}^{I}}{\det S^{I}}, & S_{22}^{IV}=\dfrac{1}{S_{22}^{I}}, \\ S_{12}^{II}=\dfrac{iS_{12}^{I}}{S_{11}^{I}}, & S_{12}^{III}=\dfrac{-S_{12}^{I}}{\det S^{I}}, & S_{12}^{IV}=\dfrac{iS_{12}^{I}}{S_{22}^{I}}, \end{array} \tag{1}$$

Here $\det S^I = S^I_{11}S^I_{22}-(S^I_{12})^2$. In the matrix element, a resonance with the only decay mode is represented by a pair of complex-conjugate poles on sheet II as the nearest singularities and a pair of conjugate zeros on sheet I at the same points of complex energy. In the 2-channel case, the mentioned formulae of analytical continuations immediately give the resonance representation by poles and zeros on the 4-sheeted Riemann surface. One must discriminate between three types of resonances described by a pair of conjugate zeros on sheet I: (**a**) in S_{11}, (**b**) in S_{22}, (**c**) in each of S_{11} and S_{22}. A resonance of every type is represented by a pair of complex-conjugate clusters (of poles and zeros on the Riemann surface) of size typical of strong interactions. Thus, we arrive at the notion of three standard pole-clusters which represent two-channel bound states of quarks and gluons. Note that this resonance division into types is not formal. In paricular, the resonance, coupled strongly with the first ($\pi\pi$) channel, is described by the pole cluster of type (**a**); if the resonance is coupled strongly with the $K\overline{K}$ and weakly with $\pi\pi$ channel (say, if it has a dominant $s\overline{s}$ component), then it is represented by the cluster of type (**b**); finally, since a most noticeable property of a glueball is the flavour-singlet structure of its wave function and, therefore, (except the factor $\sqrt{2}$ for a channel with neutral particles) practically equal coupling with all the members of the nonet, then a glueball must be represented by the pole cluster of type (**c**) as a necessary condition.

Just as in the 1-channel case, the existence of a particle bound-state means the presence of a pole on the real axis under the threshold on the physical sheet, so in the 2-channel case, the existence of a bound state in channel 2 ($K\overline{K}$ molecule), which, however, can decay into channel 1 ($\pi\pi$ decay), would imply the presence of a pair of complex conjugate poles on sheet II under the threshold of the second channel without an accompaniment of the corresponding shifted pair of poles on sheet III [5].

In our previous 2-channel analysis of the $\pi\pi$ scattering, we have obtained satisfactory description ($\chi^2/\mathrm{ndf} \approx 1.00$) with two resonances ($f_0(975)$ and $f_0(1500)$) and

with the large $\pi\pi$-background. There, in the uniformizing variable, we have taken into account only the right-hand branch-points at $s = 4m_\pi^2$ and $s = 4m_K^2$.

Now, to take also the left-hand branch-point at $s = 0$ into account, we use the uniformizing variable

$$v = \frac{m_K\sqrt{s-4m_\pi^2} + m_\pi\sqrt{s-4m_K^2}}{\sqrt{s(m_K^2-m_\pi^2)}}, \tag{2}$$

which maps the 4-sheeted Riemann surface onto the v-plane, divided into two parts by a unit circle centered at the origin. The sheets I (II), III (IV) are mapped onto the exterior (interior) of the unit disk on the upper and lower v-half-plane, respectively. The physical region extends from the point i on the imaginary axis ($\pi\pi$ threshold) along the unit circle clockwise in the 1st quadrant to point 1 on the real axis ($K\overline{K}$ threshold) and then along the real axis to point $b = \sqrt{(m_K+m_\pi)/(m_K-m_\pi)}$ into which $s = \infty$ is mapped on the v-plane. The intervals $(-\infty, -b], [-b^{-1}, b^{-1}], [b, \infty)$ on the real axis are the images of the corresponding edges of the left-hand cut of the $\pi\pi$-scattering amplitude. The type (**a**) resonance is represented in S_{11} by two pairs of the poles on the images of the sheets II and III, symmetric to each other with respect to the imaginary axis, by zeros, symmetric to these poles with respect to the unit circle.

Note that the variable v is uniformizing for the $\pi\pi$-scattering amplitude, however, the amplitudes of the $K\overline{K}$ scattering and $\pi\pi \to K\overline{K}$ process do have the cuts on the v-plane, which arise from the left-hand cut on the s-plane, starting at $s = 4(m_K^2 - m_\pi^2)$. Under conformal mapping (2), this left-hand cut is mapped into cuts which begin at the points $v = (m_K\sqrt{m_K^2 - 2m_\pi^2} \pm i m_\pi)/(m_K^2 - m_\pi^2)$ on the unit circle on the v-plane, go along it up to the imaginary axis, and occupy the latter. This left-hand cut will be neglected in the Riemann-surface structure, and the contribution on the cut will be taken into account in the $K\overline{K}$ background as a pole on the real s-axis on the physical sheet in the sub-$K\overline{K}$-threshold region; on the v-plane, this pole gives two poles on the unit circle in the upper half-plane, symmetric to each other with respect to the imaginary axis, and two zeros, symmetric to the poles with respect to the real axis, *i.e.* at describing the process $\pi\pi \to K\overline{K}$, one additional parameter is introduced, say, a position p of the zero on the unit circle.

For the simultaneous analysis of experimental data on the coupled processes it is convenient to use the Le Couteur-Newton relations [6] expressing the S-matrix elements of all coupled processes in terms of the Jost matrix determinant $d(k_1, k_2) \equiv$

$d(s)$, the real analytic function with the only square-root branch-points at the process thresholds $k_i = 0$.

On v-plane the Le Couteur-Newton relations are

$$S_{11} = \frac{d(-v^{-1})}{d(v)}, \quad S_{22} = \frac{d(v^{-1})}{d(v)}, \quad S_{11}S_{22} - S_{12}^2 = \frac{d(-v)}{d(v)} \tag{3}$$

with the d-function that on the v-plane already does not possess branch-points is taken as

$$d = d_B d_{res}, \tag{4}$$

where $d_B = B_\pi B_K$; B_π contains the possible remaining $\pi\pi$-background contribution, related to exchanges in crossing channels; B_K is that part of the $K\overline{K}$ background which does not contribute to the $\pi\pi$-scattering amplitude

$$B_K = v^{-1}(1 - pv)(1 + p^*v). \tag{5}$$

The function $d_{res}(v)$ represents the contribution of resonances, described by one of three types of the pole-zero clusters, *i.e.*, except for the point $v = 0$, it consists of zeros of clusters:

$$d_{res} = v^{-M} \prod_{n=1}^{M} (1 - v_n^* v)(1 + v_n v), \tag{6}$$

where n runs over the independent zeros; therefore, for resonances of the types (**a**) and (**b**), n has two values, for the type (**c**), four values; M is the number of pairs of the conjugate zeros.

On the basis of these formulas, we analyze simultaneously the available experimental data on the $\pi\pi$-scattering [7, 8] and the process $\pi\pi \to K\overline{K}$ [9] in the channel with $I^G J^{PC} = 0^+0^{++}$.

To obtain the satisfactory description ($\chi^2/\text{ndf} \approx 2.2$) of the s-wave $\pi\pi$ scattering from the threshold to 1.89 GeV and $|S_{12}|$ from the threshold to 1.4 GeV (where the 2-channel unitarity is valid), we have taken $B_\pi = 1$ in eq.(4), and three multichannel resonances turned out to be sufficient: the two ones of the type (**a**) ($f_0(660)$ and $f_0(980)$) and $f_0(1500)$ of the type (**c**). A satisfactory description of the phase shift of the $\pi\pi \to K\overline{K}$ matrix element is obtained to 1.5 GeV with the value of the parameter $p = 0.993613 - 0.112842i$ (this corresponds to the position of the pole on the s-plane at $s = 4(m_K^2 - m_\pi^2) - 0.06$).

In the table, the obtained parameter values of poles on the corresponding sheets of the Riemann surface are cited on the complex energy plane ($\sqrt{s_r} = \mathrm{E}_r - i\Gamma_r/2$). We

stress that these are not masses and widths of resonances. Since, for wide resonances, values of masses and widths are very model-dependent, it is reasonable to report characteristics of pole clusters which must be rather stable for various models. Let

	$f_0(660)$		$f_0(980)$		$f_0(1500)$	
Sheet	E, MeV	Γ, MeV	E, MeV	Γ, MeV	E, MeV	Γ, MeV
II	600±14	620±26	990±5	25±10	1480±15	400±35
III	720±15	6±2	984±16	200±32	1540±25	300±35
					1530±24	400±38
IV					1500±22	300±34

us indicate the constant values of the obtained-state couplings with the $\pi\pi$ and $K\overline{K}$ systems calculated through the residues of amplitudes at the pole on sheet II. Taking the resonance part of the amplitude in the form $T_{ij}^{res} = \sum_r g_{ir}g_{rj}D_r^{-1}(s)$, where $D_r(s)$ is an inverse propagator ($D_r(s) \propto s - s_r$), we obtain (we denote the coupling constants with the $\pi\pi$ and $K\overline{K}$ systems through g_π and g_K, respectively) for $f_0(660)$: $g_\pi = 0.7376 \pm 0.12$ GeV and $g_K = 0.37 \pm 0.1$ GeV, for $f_0(980)$: $g_\pi = 0.158 \pm 0.03$ GeV and $g_K = 0.86 \pm 0.09$ GeV, for $f_0(1500)$: $g_\pi = 0.347 \pm 0.028$ GeV. In this 2-channel approach, there is no point in calculating the coupling constant of the $f_0(1500)$ state with the $K\overline{K}$ system, because the 2-channel unitarity is valid only to 1.4 GeV, and, above this energy, there is a considerable disagreement between the calculation of the amplitude modulus S_{12} and the experimental data.

Let us indicate also scattering lengths calculated in our approach. For the $K\overline{K}$ scattering, we obtain $a_0^0(K\overline{K}) = -0.932 \pm 0.11 + (0.706 \pm 0.09)i, \quad m_{\pi^+}^{-1}$. A presence of the imaginary part in $a_0^0(K\overline{K})$ reflects the fact, that already at the threshold of the $K\overline{K}$ scattering, other channels ($2\pi, 4\pi$ etc.) are opened.

For the $\pi\pi$ scattering, we obtain: $a_0^0 = 0.27 \pm 0.06, \quad m_{\pi^+}^{-1}$. Compare with results of some other works both theoretical and experimental: the value 0.26 ± 0.05 (L. Rosselet et al.[8]), obtained in the analysis of the decay $K \to \pi\pi e\nu$ with using Roy's model; 0.24 ± 0.09 (A.A. Bel'kov et al.[8] from analysis of the process $\pi^- p \to \pi^+\pi^- n$ with using the effective range formula; 0.23 (S. Ishida et al.[**?**], modified approach to analysis of $\pi\pi$ scattering with using Breit-Wigner forms; 0.16 (S. Weinberg [10], current algebra (non-linear σ-model)); 0.20 (J. Gasser, H. Leutwyler [10], the theory with the non-linear realization of chiral symmetry); 0.26 (M.K. Volkov [10], the theory with the linear realization of chiral symmetry).

So, an existence of the low-lying state with the properties of the σ-meson and the obtained value of the $\pi\pi$-scattering length seem to suggest the linear realization of chiral symmetry.

Here, a parameterless description of the $\pi\pi$ background in the channel with $I^G J^{PC} = 0^+0^{++}$ is first given.

The $f_0(1500)$ state is represented by the pole cluster on the Riemann surface of the S-matrix which corresponds to a flavour singlet, *e.g.* the glueball.

Finally, a minimum scenario of the simultaneous description of the processes $\pi\pi \to \pi\pi, K\overline{K}$ in the channel with $I^G J^{PC} = 0^+0^{++}$ does not require the $f_0(1370)$ resonance; therefore, if this meson exists, it must be weakly coupled with the $\pi\pi$ channel, *e.g.* be the $s\bar{s}$ state (as to that assignment of the $f_0(1370)$ resonance, we agree with the work [11]).

This work has been supported by the Grant Program of Plenipotentiary of Slovak Republic at JINR. Yu.S. and M.N. were supported in part by the Slovak Scientific Grant Agency, Grant VEGA No. 2/7175/20; and D.K., by Grant VEGA No. 2/5085/99.

References

[1] D. Krupa, V.A. Meshcheryakov, Yu.S. Surovtsev, *Nuovo Cimento* **A 109**, 281 (1996).

[2] Review of Particle Physics, *Europ. Phys. J.* **C3**, 1 (1998).

[3] Y. Nambu, G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961); M.K. Volkov, *Ann. Phys.* **157**, 282 (1984); T. Hatsuda, T. Kunihiro, *Phys. Rep.* **247**, 223 (1994); R. Delbourgo, M.D. Scadron, *Mod. Phys. Lett.* **A10**, 251 (1995).

[4] B.S. Zou, D.V. Bugg, *Phys. Rev.* **D48**, R3948 (1993); **D50**, 591 (1994); M. Svec, *Phys. Rev.* **D53**, 2343 (1996); S. Ishida et al., *Progr. Theor. Phys.* **95**, 745 (1996); **98**, 621 (1997); N. Törnqvist, *Phys. Rev. Lett.* **76**, 1575 (1996); R. Kamiński, L. Leśniak, B. Loiseau, hep-ph/9810386 (1999).

[5] D. Morgan, M.R. Pennington, *Phys. Rev.* **D48**, 1185 (1993).

[6] K.J. Le Couteur, *Proc. Roy. Soc.* **A256**, 115 (1960); R.G. Newton, *J. Math. Phys.* **2**, 188 (1961); M. Kato, *Ann. Phys.* **31**, 130 (1965).

[7] B. Hyams et al., *Nucl. Phys.* **B64**, 134 (1973); ibid., **B100**, 205 (1975).

[8] A. Zylbersztejn et al., *Phys. Lett.* **B38**, 457 (1972); P. Sonderegger, P. Bonamy, Proc. 5th Intern. Conf. on Elementary Particles, Lund, 1969, paper 372; J.R. Bensinger et al., *Phys. Lett.* **B36**, 134 (1971); J.P. Baton et al., *Phys. Lett.* **B33**, 525, 528 (1970); P. Baillon et al., *Phys. Lett.* **B38**, 555 (1972); L. Rosselet et al., *Phys. Rev.* **D15**, 574 (1977); A.A. Kartamyshev et al., Pis'ma v Zh. Eksp. Teor. Fiz. **25**, 68 (1977). A.A. Bel'kov et al., *Pis'ma v Zh. Eksp. Teor. Fiz.* **29**, 652 (1979).

[9] A.B. Wicklund et al., *Phys. Rev. Lett.* **45**, 1469 (1980); D. Cohen et al., *Phys. Rev.* **D22**, 2595 (1980); A. Etkin et al., *Phys. Rev.* **D25**, 1786 (1982).

[10] S. Weinberg, *Phys. Rev. Lett.* **17**, 616 (1966); B.W. Lee, H.T. Nieh, *Phys. Rev.* **166**, 1507 (1968); J. Gasser, H. Leutwyler, *Ann. Phys.* **158**, 142 (1984); M.K. Volkov, Physics of Elem. Particles and Atom. Nuclei **17** part 3, 433 (1986).

[11] L.S. Celenza, Huangsheng Wang, C.M. Shakin, *Preprint of Brooklyn College of the City Univ. of New York*, BCCNT: 00/041/289, April 2000.